CREEP IN MATERIALS WITH DIFFERENT TENSION AND COMPRESSION BEHAVIOR

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Experimental data on creep in tubular specimens loaded in tension and in compression are cited. It is demonstrated experimentally that the resulting vector of the creep strain rates is orthogonal to the curve of constant scattering intensity. Approximate analytical formulas making it possible to describe the creep process undergone by materials having different behavior patterns in tensile and compressive loading are derived.

1. Light alloys exhibiting reasonably stable properties in the sense of instantaneous elastoplastic characteristics, these latter being possibly independent both of the direction of the applied load and of its sign, manifest essential anisotropy in their creep behavior. The strain rate at a given specified stress may vary several times in magnesium alloys, titanium alloys, and the like, depending on the direction and sign of the load. For example, the instantaneous $\sigma-\varepsilon$ diagram taken under compressive loading and tensile loading at a temperature of 200°C, in the case of aluminum-magnesium alloy, coincides completely both in the elastic region and in the plastic region, and the creep strain rates η differ by more than three times at the same stress levels in compressive loading and tensile loading.

Figure 1 shows the results of experiments conducted with coupons cut from a cylindrical blank of aluminum-magnesium alloy at the temperature 200°C. Here the stress σ is plotted as abscissa in kg_{force} mm² units, and the natural logarithms of the creep strain rates η are plotted in reciprocal hour units as ordinate. The numerals on the curve indicate data taken on coupons cut at a 45° angle to the axis of the cylindrical stock (curve 1), cut from the diametral plane of the stock (curve 2), and cut from the axial direction of the stock (curve 3). The hollow dots correspond to experimental data points referable to tensile loading. The numeral 4 indicates experimental data taken in tensile loading of coupons cut from the axial direction of the blank. It is clear in Fig. 1 that the material is anisotropic in its creep properties, and exhibits different degrees of strength under tensile and compressive loading.

The creep process experienced by this material at 200°C and at stresses starting with 8 kg/mm² on up takes place practically without any hardening, and can be described for any direction ν by a formula of the type

$$|\eta| = e^{-K_v + \beta_v |\sigma|} \tag{1.1}$$

The values of K_{ν} and β_{ν} can be determined with ease, from the diagram in Fig. 1, for each of the above directions under tensile and compressive loading

$$\beta_1 = 1.24, \quad \beta_2 = 1.0, \quad \beta_3 = \beta_4 = 0.75 [\,\mathrm{mm}^2/\mathrm{kg}] \\ K_1 = 16.2, \quad K_2 = 14.6, \quad K_3 = 13.0, \quad K_4 = 14.1$$
(1.2)

Here the numerals subscripted to the letters correspond to the numbers appearing in Fig. 1.

The calculated curves plotted on the basis of Eq. (1.1) with the characteristics (1.2) exhibit a reasonable approximation to the experimental creep curves for all four series of experiments.

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With the object of analyzing the behavior of this material under conditions of a complex stressed state, an extended series of experiments was undertaken under the same temperature conditions, using tubular coupons cut from the axial direction on the original experimental blank (effective length of tested coupon 80 mm, outer and inner diameters respectively 20 mm and 18 mm), in different combinations of axial tensile loading and bending moment applied to the test coupon. In purely axial tensile loading, the results for the tubular coupons were found to match the results obtained with small cylindrical coupons on which the original characteristics of the material (1.2) were determined.

Points at which the creep processes take place at the same intensity, in the sense of the dissipation intensity $W = \sigma_{ij}\eta_{ij}$, were found experimentally on the stress plane $\sigma\tau$. Three experimental curves $W = \text{const corresponding to different levels of intensity of the process were plotted. The stress levels <math>\sigma$ and τ , in kg/mm² units, at which the experiments were conducted, the axial creep stress rates, the shear strain rates $d\gamma/dt = 2\eta_{12}$ measured in the course of the experiment, and the experimental values of $W = \sigma\eta_{11} + \tau 2\eta_{12}$, are all tabulated (see Table 1).

Figure 2 shows one of the W = const curves, plotted as a continuous curve, with circles indicating the stressed state at which the experiment was performed. At each of the indicated points, with double the value of η_{11} plotted horizontally and the value of $2\eta_{12}$ plotted vertically, the resultant vector of the creep strain rates, which fits quite closely to the direction of the normal to the W = const curve, was constructed. A similar pattern is obtained in the case of the other two W = const curves. We realize from Fig. 2 that a surface coinciding with the W = const surface, to which the resultant vector of the creep strain rates is orthogonal, exists in the stress space, despite the complexities in the behavior of the material experiencing creep.

This result stands as an excellent and compelling confirmation of the hypothesis that a potential function exists for the rates of creep strains in the case of anisotropic media.

σ	T	ri*11103	m11103	2n*12103	2ni2103	W103
$\begin{array}{c} 8.00\\ 7.76\\ 7.20\\ 5.77\\ 3.72\\ 1.93\\ 0\\ 9.00\\ 8.70\\ 8.00\\ 6.33\\ 4.00\\ 2.10\\ 0\\ 10.00\\ 9.60\\ 8.80\\ 6.90\\ 4.30\\ 2.22 \end{array}$	$\begin{array}{c} 0\\ 0.97\\ 1.80\\ 2.88\\ 3.72\\ -3.85\\ 3.91\\ 0\\ 1.09\\ 2.00\\ 3.17\\ 4.00\\ 4.20\\ 4.25\\ 0\\ 1.20\\ 2.20\\ 3.45\\ 4.30\\ 4.45 \end{array}$	$\begin{array}{c} 0.93\\ 0.91\\ 0.88\\ 0.70\\ 0.46\\\\ 1.96\\ 1.92\\ 1.84\\ 1.51\\ 0.87\\\\ 4.15\\ 3.90\\ 3.80\\ 3.10\\ 1.71\\\\\\\\\\\\\\\\\\\\ -$	$\begin{array}{c} 0.95\\ 0.90\\ 0.85\\ 0.65\\ 0.36\\ 0.18\\ \hline 1.95\\ 1.95\\ 1.95\\ 1.84\\ 1.55\\ 0.70\\ 0.20\\ \hline 4.10\\ 3.70\\ 3.50\\ 3.50\\ 3.00\\ 1.25\\ 0.45\\ \end{array}$	$\begin{array}{c} - \\ 0.35 \\ 0.68 \\ 1.10 \\ 1.41 \\ - \\ - \\ 0.90 \\ 1.64 \\ 2.56 \\ 2.78 \\ - \\ - \\ 2.06 \\ 3.80 \\ 5.70 \\ 5.83 \\ - \end{array}$	$\begin{array}{c} 0.40\\ 0.70\\ 1.20\\ 1.70\\ 1.75\\ 1.90\\ \hline \\ 0.65\\ 1.50\\ 2.57\\ 3.70\\ 4.20\\ 4.15\\ \hline \\ 1.50\\ 3.34\\ 5.24\\ 7.30\\ 8.40\\ \end{array}$	$\begin{array}{c} 7.60\\ 7.40\\ 7.35\\ 7.20\\ 7.66\\ 7.10\\ 7.45\\ 17.55\\ 17.71\\ 17.63\\ 17.85\\ 17.60\\ 18.10\\ 17.64\\ 41.00\\ 37.40\\ 37.40\\ 37.94\\ 38.45\\ 36.77\\ 38.40\\ \end{array}$
0	4.58			-	9.00	41.20

TABLE 1

2. It would be of interest to provide some at least approximate analytical dependences which would be helpful in describing the creep process in the case of media exhibiting different tensile strength and compressive strength. While remaining within the framework of the hypothesis stating the existence of a potential function of the creep strain rates, we find it clearly insufficient to assume that this potential function would depend solely on the quadratic form of the stress and anisotropy tensors, and that odd invariants have to be introduced into consideration.

We now introduce the simplest assumption in this context: we assume that the creep process depends on the sign of the first invariant of the stress tensor $I_1 = \sigma_{ii}$, i.e., that the entire stress space is broken up by the $\sigma_{ii} = 0$ plane into two subspaces in each of which we use the introduction of the potential function dependent solely on the quadratic invariants with their characteristics in an attempt to describe the creep process. Or, using geometric parlance, we suppose that the family of potential surfaces $\Phi = \text{const}$ in the space of the principal stresses constitutes two families of coaxial cylinders (in the general case of anisotropic media: not circular cylinders necessarily) with axes inclined at the same angles to the coordinate axes. Both of these families are adjoined in the vicinity of the deviator plane passing through the origin of coordinates. The vectors of the creep strain rates are orthogonal to those surfaces.

In the case of the stressed state constituting a combination of tension and torsion $I_1 \ge 0$, so that the characteristics of the material corresponding to compression do not show up in the potential function. In order to determine the quadratic form in the potential function with attention given to the axial symmetry of the material, it is sufficient to have the first three pairs of characteristics. Making use of the condition that the top three curves corresponding to the tensile loading data in Fig. 1 intersect at the point $\sigma_0 = 6.5$ kg/mm², and repeating arguments similar to those put forward in [1], we construct our potential function in the form

$$\eta_{ij} = \partial \Phi / \partial \sigma_{ij}, \ \Phi = M \left(S/T \right)^{1/2} \exp \left\{ - \left[D + \sigma_0 \left(2T/S \right)^{1/2} \right] + T^{1/2} \right\} \\S = 3\sigma_{ij} \sigma_{ij} \sigma_{ij} \sigma_{ij} = \sigma_{ij} - \frac{1}{3} \delta_{ij} \sigma_{kk} \\T = A_{11} \left(\sigma_{22} - \sigma_{33} \right)^2 + A_{22} \left(\sigma_{33} - \sigma_{11} \right)^2 + A_{33} \left(\sigma_{11} - \sigma_{22} \right)^2 \\+ 2A_{12} \sigma_{12}^2 + 2A_{23} \sigma_{23}^2 + 2A_{31} \sigma_{31}^2$$

$$(2.1)$$

where M is a constant having the dimensionality of a reciprocal hour.

In the context of a description of creep processes occurring under combined tensile and torsional load, the quadratic form T, with due attention given to the fact that $A_{12} = A_{13}$, and with the tangential stress τ introduced into consideration, acquires the form

$$T = (A_{22} + A_{33}) \sigma^2 + 2A_{12}\tau^2 \tag{2.2}$$

the coefficients of which are found in terms of the quantities (1.2) [1]

$$A_{22} + A_{33} = \beta_3^2, \ 2A_{12} = (2\beta_1)^2 - \beta_2^2, \ D = K_{\nu} - \beta_{\nu}\sigma_0 + \frac{1}{2} \ln 2$$
(2.3)

where K_{ν} , β_{ν} is any of the first three pairs (1.2). Substitution of the values (1.2) into Eqs. (2.3) yields

$$A_{22} + A_{33} = 0.56, \quad 2A_{12} = 5.2, \quad D = 8.45$$
 (2.4)

The components of the creep strain rates from Eqs. (2.1), with Eqs. (2.2) and (2.4) taken into cognizance, will be expressed as

$$\eta_{11} = C \left(2B_1 + 0.56B_2 \right) \sigma, \qquad 2\eta_{12} = C \left(6B_1 + 5.2B_2 \right) \tau \tag{2.5}$$

Here we have

$$C = \exp \left\{ (0.56\sigma^2 + 5.2\tau^2)^{\frac{1}{2}} - 8.45 - 6.5 (0.56\sigma^2 + 5.2\tau^2)^{\frac{1}{2}} (\sigma^2 + 3\tau^2)^{-\frac{1}{2}} \right\}$$
$$B_1 = \left[(2\sigma^2 + 6\tau^2) (0.56\sigma^2 + 5.2\tau^{\frac{1}{2}}) \right]^{-\frac{1}{2}} + \sqrt{2} 6.5 (2\sigma^2 + 6\tau^2)^{-1}$$
$$B_2 = \left[(2\sigma^2 + 6\tau^2)^{\frac{1}{2}} - \sqrt{2} 6.5 \right] (0.56\sigma^2 + 5.2\tau^2)^{-\frac{1}{2}} - (2\sigma^2 + 6\tau^2)^{\frac{1}{2}} (0.56\sigma^2 + 5.2\tau^2)^{-\frac{3}{2}}$$

The values of the rates of axial creep strains η_{11}^* and shear creep strains $2\eta_{12}^*$ computed on the basis of Eqs. (2.5) are tabulated above (Table 1).

The dissipation intensity $W = \sigma \eta_{11} + \tau 2 \eta_{12}$ is obtained directly from Eqs. (2.1)

$$W = M (S)^{1/s} \exp \{-[D + \sigma_0 (2T / S)^{1/s}] + T^{1/s}\}$$
(2.6)

The shape of the curve $W^* = \text{const}$, which passes through the point $\sigma = 8 \text{ kg/mm}^2$, $\tau = 0$, and is calculated on the basis of Eq. (2.6) with the characteristics (2.4), is shown by the dashed curve in Fig. 2. In most of the experimentally investigated region of stresses, the predicted curve coincides with the experimental curve. Direct calculations show that the $\Phi = \text{const}$ curves (2.1) and $W^* = \text{const}$ curves (2.6) passing through any one point on the σ , τ plane lie quite close to each other; the greatest difference in stress levvel between them is of the order of 3%, while there is virtually no difference in the directions of the normals to those curves.

Comparison of the tabular predicted rates and experimentally measured rates of creep strains and position of the predicted $W^* = \text{const}$ curves and the curves plotted experimentally in Fig. 2 allows us to infer that the dependence (2.1) provides a completely satisfactory description of the creep process experienced by the material, even though the material exhibits different properties under tensile load and under compressive load. An exception is the region directly adjacent to a stressed state of the pure shear type when $I_1 = 0$. The result argues in favor of the hypothesis proposed at the outset as to the possibility of a separate and independent description of creep processes in the case of positive and negative values of the first invariant of the stress tensor. The greatest error in the dependence should be anticipated in the neighborhood of the zero value of I_1 .

LITERATURE CITED

1. O. V. Sosnin, ⁿAnisotropic creep of materials undergoing hardening, ⁿ MTT, No. 4 (1968).